

Introduction

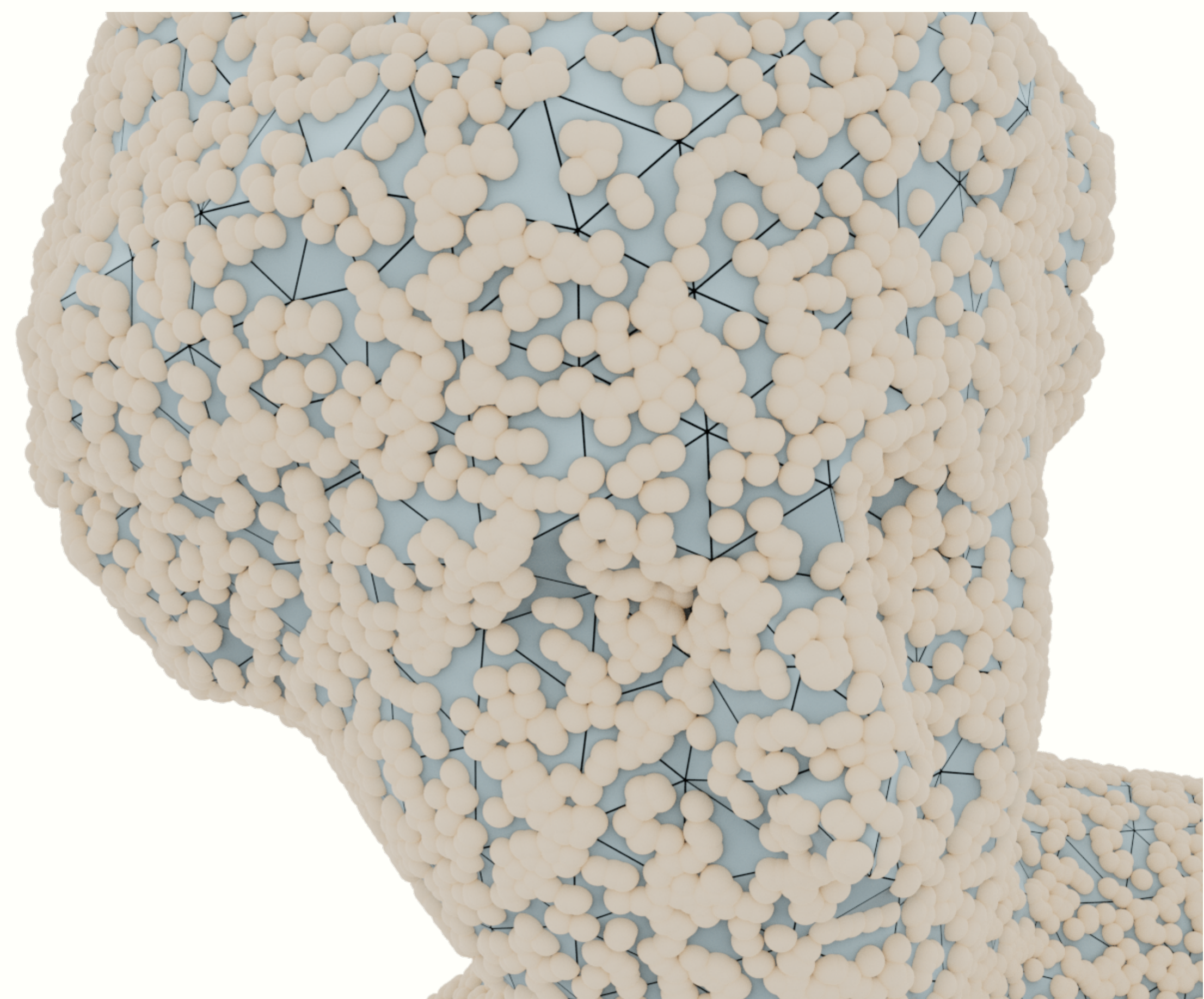
Sampling is a ubiquitous part of geometrical deep learning that operates with meshes. It allows the computation of geometric loss by converting a set of triangles into point-cloud.

Such a sampler must be

- Fast
- Differentiable
- Robust and producing low-variance estimate

Limitation of Random Uniform Sampler (RUS)

- Points are drawn idependently.
- Sampling patterns are subject to clustering.
- Distance estimate with a low number of points are subject to a large variance.



Optimization on a measure space

The triangle mesh T can be written as a measure μ_c^T carried by a union of simplexes

$$\mu_c^T(B) = \frac{1}{|T|} \sum_{t_i \in T} \int_{B \cap t_i} d\mathcal{H}^2(x).$$

Supervised learning problem

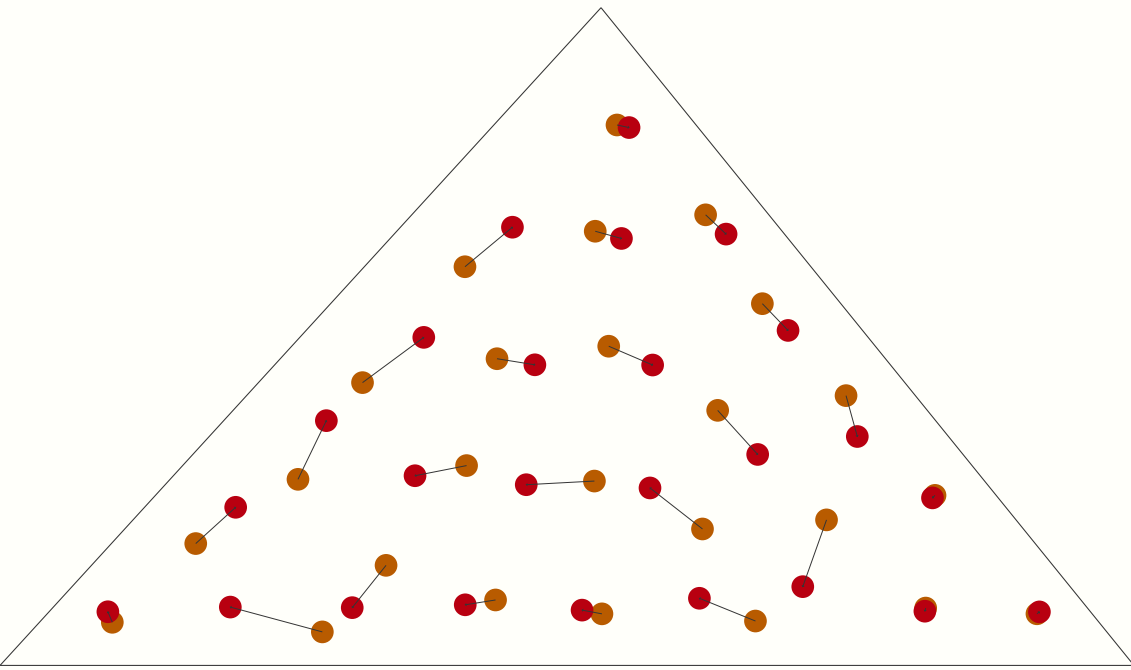
Given a sampling S and ℓ sampled points, MongeNet $\mathbf{f}_\theta(t, \ell, p)$ minimizes:

$$\mathcal{L}(t, \ell, p, S) = \underbrace{W_2^\varepsilon(\mathbf{f}_\theta(t, \ell, \mathbf{p}), S)}_{\text{fidelity}} - \alpha \underbrace{W_2^\varepsilon(\mathbf{f}_\theta(t, \ell, \mathbf{p}), \mathbf{f}_\theta(t, \ell, \mathbf{p}'))}_{\text{diversity}}$$

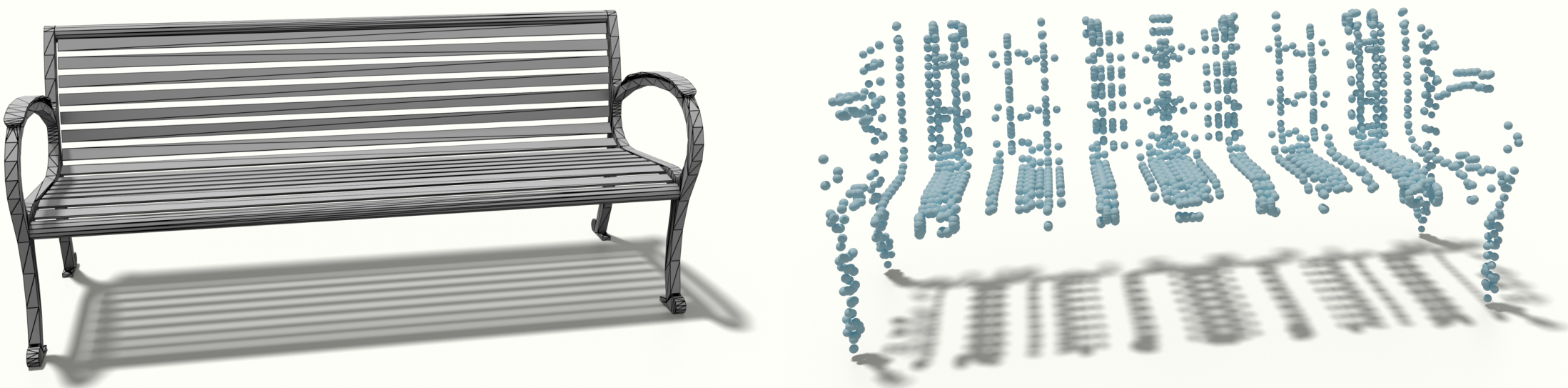
with W_2^ε the ε -regularized optimal transport [1, 2] and $\mathbf{p}, \mathbf{p}' \sim \mathcal{N}(0, 1)$, and \mathbf{f}_θ a MLP.

Encouraging entropic samples

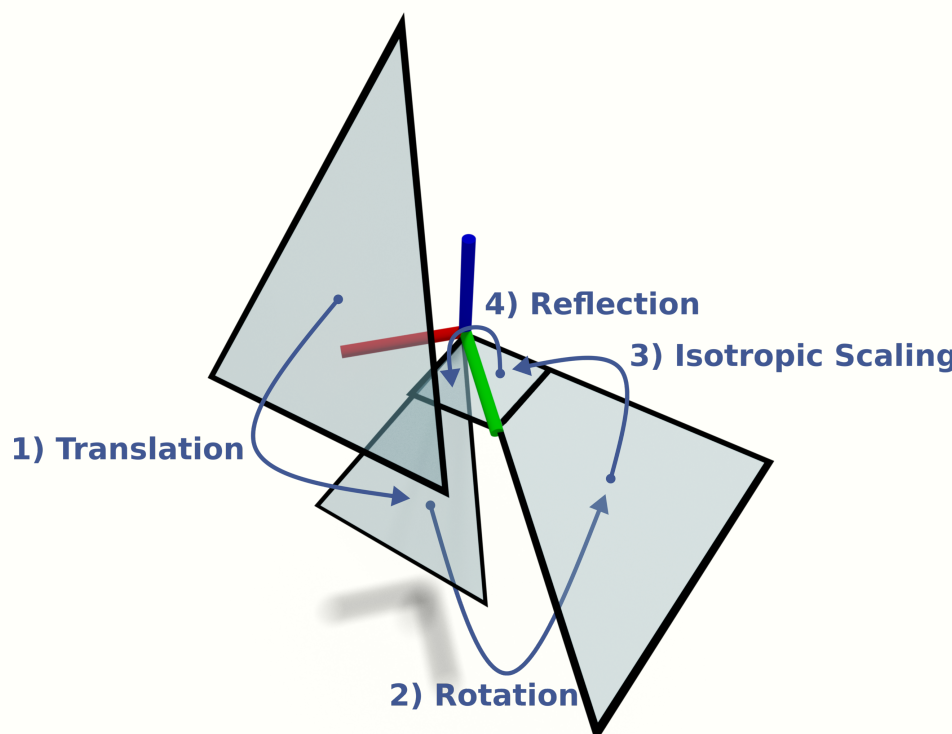
The point generation is conditioned by \mathbf{p} . Two different \mathbf{p}, \mathbf{p}' result in a different sampling pattern.



Deterministic sampling generates structured patterns



Dimensionality reduction of the learning problem



We project the triangle of \mathbb{R}^3 on a canonical space with angle preserving transformation to reduce the complexity of the learning task.

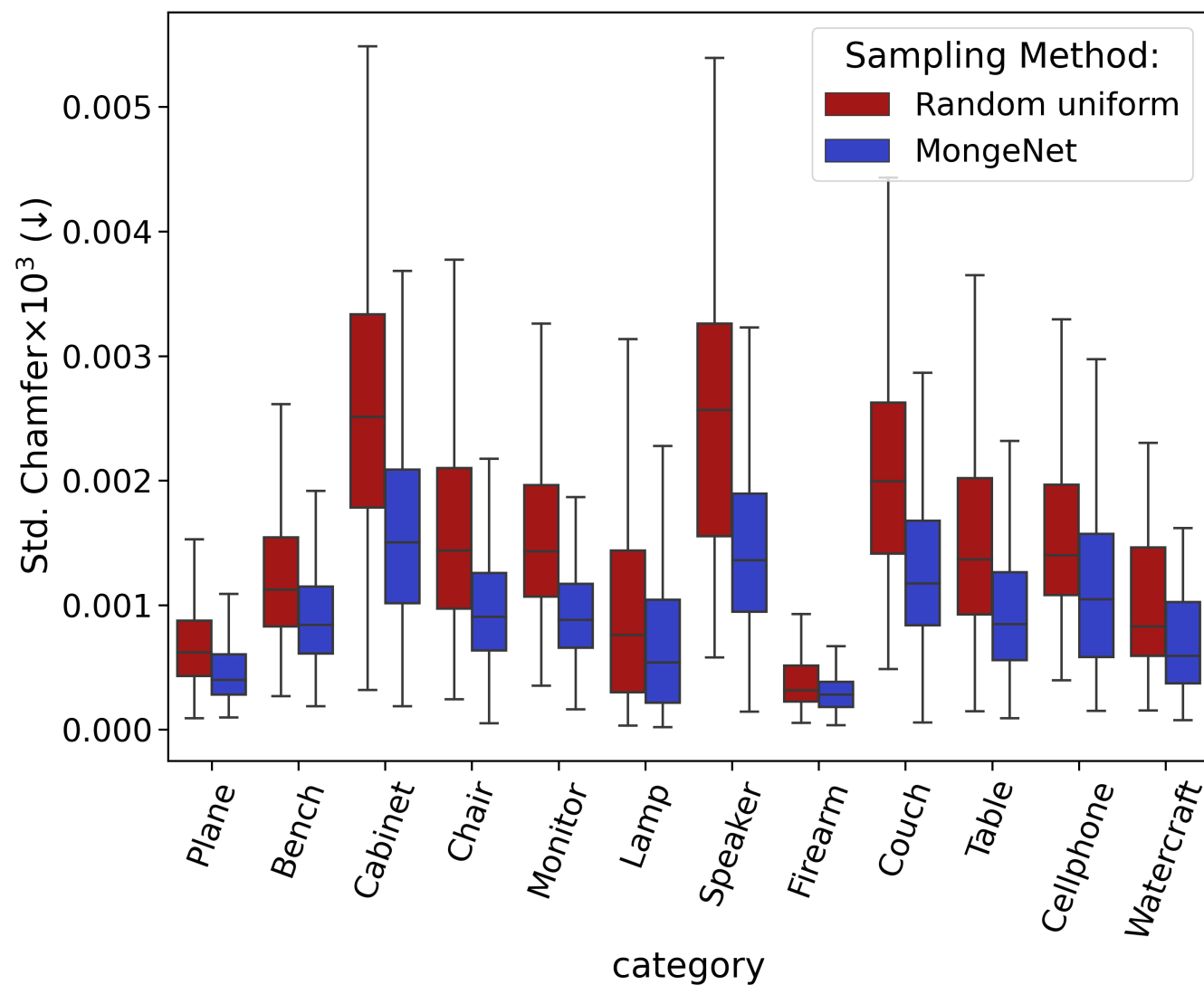
Runtime vs. Pytorch3D

# Faces	10k	20k	30k	40k	60k	80k
RUS	1.14 ms	1.50ms	1.53ms	1.52ms	1.53ms	1.53ms
MongeNet	2.89 ms	5.41 ms	7.90 ms	10.5 ms	16.0 ms	21.7 ms

Applications

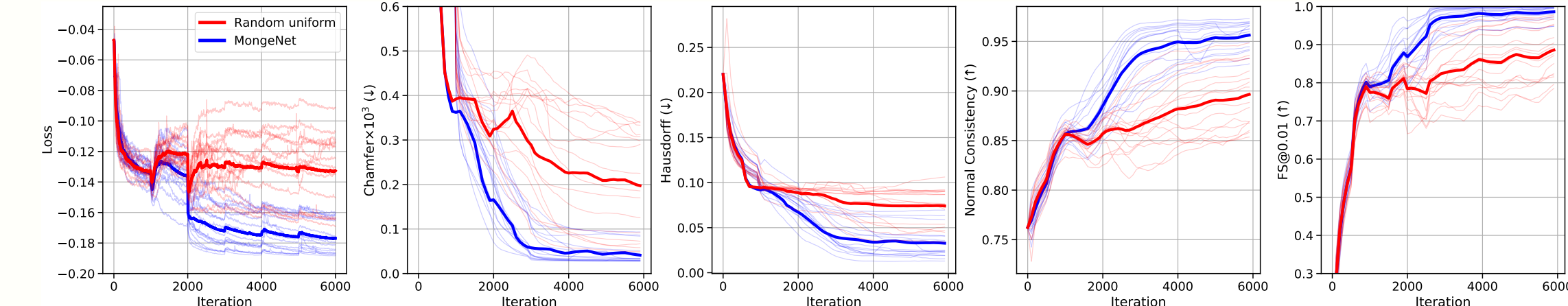
We establish the usefulness of the proposed method

For metric evaluation



MongeNet reduces the evaluation variance.

In a learning context with Point2mesh [3]



MongeNet allows training geometric deep learning models better and faster.

Easy to use

```
from src.mesh_sampler import MeshSampler
mesh_sampler = MeshSampler(mongenet, num_sampled_points, compute_normals, bs).to('cuda')
points, face_ids, normals = mesh_sampler(vertices, faces, lengths)
```

Code repo

<https://github.com/lebrat/MongeNet>

Contact us

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References

- [1] Marco Cuturi. Sinkhorn distances: Lightspeed computation of optimal transport. In *Advances in neural information processing systems*, pages 2292–2300, 2013.
- [2] Jean Feydy, Joan Glaunès, Benjamin Charlier, and Michael Bronstein. Fast geometric learning with symbolic matrices. *Advances in Neural Information Processing Systems*, 33, 2020.
- [3] Rana Hanocka, Gal Metzger, Raja Giryes, and Daniel Cohen-Or. Point2mesh: A self-prior for deformable meshes. *ACM Trans. Graph.*, 39(4), 2020.